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# Research on forecasting model of sandstone deformation rate during sine wave loading segment under the lag effect

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To reduce data variance caused by individual differences of different samples, a new experimental method is proposed by loading and unloading the axial forces with different frequencies and different waves of a certain load amplitude to the same rock sample without damaging it. Lag time segments are defined and fractionized into segments I and II. Criterions for segmentation, definitions of relevant parameters and empirical analysis are also offered afterwards. In the course of sinusoidal loading, the serious peak value dislocation of the displacement variation rate and the loading rate is defined as peak dislocation. Meanwhile, the definition of the apparent tangent modulus is put forward and the linear relation between it and the vertical force in the frequency of 0.1, 0.2, 0.5 Hz sinusoidal loading segment is confirmed to be ever-present on the basis of the test data. Then the calculating formula of the deformation rate in non-lag time is deduced. It is thus suggested that the deformation rate should be codetermined by the loading rate df/dt and instant load $f(t)$, which well explains the peak dislocation of the time-variable curve peak value of $dl/dt$ and deformation rate of $df/dt$. Finally the lag time derivation model is established and by comparing the calculated values with the measured ones, it is demonstrated that the above formula offers a better simulation of the sandstone deformation rate in the sinusoidal loading segment, with the load amplitude being 96 kN and the frequency ranging from 0.1 Hz to 0.5 Hz.

**lag time segments, peak shifting phenomenon, apparent tangent modulus, deformation rate**

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## 1 Introduction

Quite a number of important achievements have been made in the research of rock sample mechanical response under cyclic loading, but much can be done to bring the study on the hysteresis effect and hysteretic circle simulation to a further-developed level. In 1968, Gordon and Davis[1] reported the hysteresis effect of the saturated rock under cyclic loading for the first time. In 1981, Spencer[2] discovered that the stress relaxation existed in the saturated rock. Holcomb[3] studied the memory, relaxation and damage of the saturated rock by the diabase and granite. In China, Xi et al.[4–6] introduced a macroscopic model which was made of hysteretic nonlinear elastic material, called Preisach-Mayergoyz model (PM model) to describe these characteristics of the rock. In 2005 Xu et al.[7–9] found that

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the water content had little influence on the elastic after-effect phenomenon of the rock deformation. No complete definition, division of hysteretic time segment, or determination of relative parameters and example analysis based on the ms time segment measurement could be found in the above research findings. Meanwhile, in the elastic modulus analysis during the cyclic process, the hysteretic-time-effect segment is usually removed, and only the hysteretic-time-effect segment is taken to make the homogenization calculation. The variation of the tangent modulus in terms of the time effect in the whole loading segment is not taken into account.

As for the hysteretic circle simulation, McCall and Guyer[10] presented that the quasi-static state equation and the elastic wave response were controlled by the micro-elastic element in 1994; Tutuncu et al.[11, 12] analyzed some main factors influencing nonlinear elastic behavior through test in 1998. Li[13] pointed out that the hysteretic circle of the viscoelastic model was a curve from the origin gradually close to an ellipse. These literatures all lay particular stress on proceeding with geometric aspect, and use a single curve form to simulate the hysteretic circle, so it would make the physical meaning of simulation parameters not clear, and the applicability of simulation with different load patterns cannot be ensured. In addition, Ge and Lu[14] obtained a lot of related research achievements through rock fatigue damage and destruction test.

Therefore, with the relation between instantaneous deformation rate and instantaneous loading rate obtained, the instantaneous deformation rate in the micro time segment can be accurately simulated based on the known load type, then the stress-strain diagram of the rock under the cyclic loading would be forecasted, laying the foundation for accurately analyzing the energy conversion progress under the cyclic loading. To start from the prediction of deformation rate in the sine wave loading segment measured based on ms time segment, the prediction of the whole loading and unloading segments and energy conversion are underpinned.

## 2 Test instrument and scheme

The RMT-150C rock mechanics testing system developed by Wuhan Institute of Rock and Soil Mechanics, Chinese Academy of Sciences is used as the main text platform (Figure 1), and the sandstone gathered at a construction site in China Three-gorges University is taken as the research project. Water drilling is adopted as the sampling mode, and the test diameter is 54 mm, with the height of 100 mm. To reduce the data discretization caused by the individual difference of samples, an axial force cycle is put forward on a rock sample without destruction, and the force cycle is under a certain loading amplitude with different waveforms and different frequencies. At first, the text is treated with multi-stage preloading compaction. When the loading amplitude reaches to 96 kN, in the order of first triangular wave then sine wave, and from low frequency to high frequency, these are applied in sequence: 0.1 Hz triangular wave, 0.2 Hz triangular wave, 0.5Hz triangular wave, 0.1 Hz sine wave, 0.2 Hz sine wave, 0.5 Hz sine wave. Then attention is focused on the analysis and simulation of the text data of sine wave unloading segment in the following research.

**Figure 1** RMT-150C rock mechanics testing system.

## 3 Definition of hysteretic time segment and "peak inconsistent" phenomenon

When the positive and negative variation appears in the loading rate, the displacement change rate $\frac{\mathrm{d}l}{\mathrm{d}t}$ and the force loading rate $\frac{\mathrm{d}f}{\mathrm{d}t}$ are obviously out of step, then this stage is defined as the hysteretic time segment. And it can be divided into segment I and segment II.

Segment I. The starting point is 0, the ending point is under the condition $\frac{\mathrm{d}l}{\mathrm{d}t}=0$. In this segment, $\frac{\mathrm{d}f}{\mathrm{d}t}\times\frac{\mathrm{d}l}{\mathrm{d}t}\leqslant 0$, $\frac{\mathrm{d}f}{\mathrm{d}t}>0$, $\frac{\mathrm{d}l}{\mathrm{d}t}\leqslant 0$.

Segment II. The starting point is under the condition $\frac{\mathrm{d}l}{\mathrm{d}t}=0$, the ending point is under the condition $\left(\frac{\mathrm{d}l}{\mathrm{d}t}\right)_{\max}$. In this segment, $\frac{\mathrm{d}f}{\mathrm{d}t}\times\frac{\mathrm{d}l}{\mathrm{d}t}\geqslant 0$, the changing rate of $\frac{\mathrm{d}l}{\mathrm{d}t}$ and $\frac{\mathrm{d}f}{\mathrm{d}t}$ is obviously not in sync.

In order to delay the establishment of numerical model about time, $t_z$ is defined as the first lag of time. $t_f$ is defined as the second lag of time. $l_y$ is the starting value displacement at the uniform section. In Figure 2, the lag of time is $t_z$=351 ms. After the lag time, the time of reaching peak is $t_f$=404 ms, the starting value displacement at the uniform section is $l_y = 0.145 \times 10^{-3} \times 100 = 0.0145$ (mm).

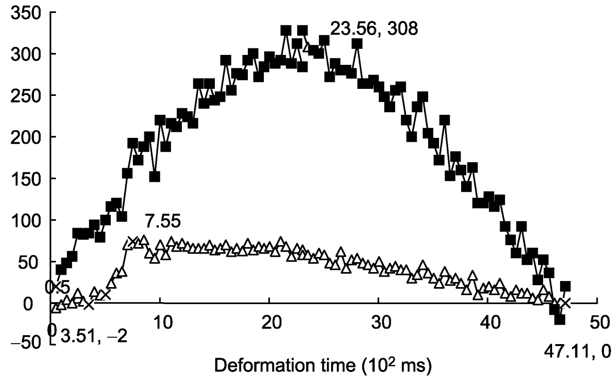


**Figure 2** Lag time in loading the sine-wave when the peak load of 0.1 Hz sandstone is 96 kN, −△−, Vertical displacement difference/time difference (mm/s); −■−, vertical force difference/time difference ($10^{-5}$ kN/ms).

From Figure 2, we can conclude that the changing rate of displacement and the loading rate of peak force are of serious dislocation. The difference of the time of reaching peak can reach to (23.56−7.55)×100=1600 ms. Here we define the serious dislocation between the changing rate of displacement and the loading rate of peak force in the process of loading the sine-wave as a phenomenon called "peak shifting", the reasons for the emergence will be given in the following analysis.

## 4 Apparent tangent modulus and its relationship with the vertical force

First, the definition of apparent tangent modulus $E(t)$ is

$$(\sigma_{t+\nabla t}-\sigma_t)/(\varepsilon_{t+\nabla t}-\varepsilon_t)=\nabla\sigma/\nabla\varepsilon=E(t), \qquad (1)$$

where σ is the axis stress, ε is the axis strain, $t$ and $t+\nabla t$ are two adjacent measurement points, $\nabla t$ is the interval between measurements. In this experiment, $\nabla t$ =50 ms, $E(t)$ is the connection slope between the two adjacent measuring points.

In the cyclic loading process when the peak load of 0.1 Hz sandstone is 96 kN, there is an obvious correlation between the magnitude of apparent tangent modulus and the vertical force in loading the sine-wave, and a good consistency in fitting the linear relationship is obtained (see Figure 3). It shows that under this waveform, the linear relationship of the magnitude of apparent tangent modulus and the vertical force in the load section which is impacted by non-lag time always exists, and the coefficient is almost the same.

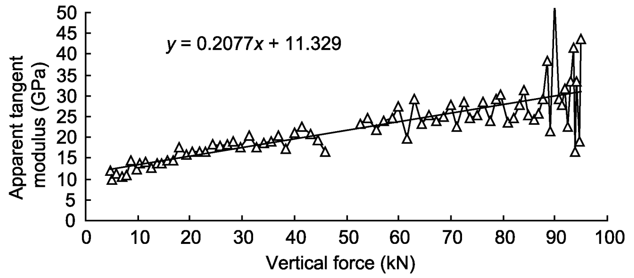


**Figure 3** Linear fit between the apparent tangent modulus (GPa) and the vertical force in loading the sine-wave of sandstone.

$E(t)=af(t)+b$ , here $a$ = 0.2077, $b$=11.329, the unit of apparent tangent modulus $E(t)$ is GPa, the unit of the vertical force is kN. $a$ is the change rate of the apparent tangent modulus with load. The greater the $a$, the higher the growth rate of the apparent tangent modulus under the same load increment will be. When the load increases, the specimen compaction will be more adequate. $b$ is the corresponding size of the apparent tangent modulus when the load tends to 0.

As we can see from Table 1 that the coefficient $a$ continuously declines with the gradual increase in loading frequency sine wave. It shows that when the loading frequency increases, the loading rate will increase rapidly. Deformation of rock samples under the same load is not sufficient, which leads to insufficient compaction of the rock sample in the loading process. However, the change of reduction rate is not obvious. When the loading rate increases by five times, the reduction rate is still less than 10%.

## 5 Research on displacement rate formula

### 5.1 Establishment of the formula

From formula (1), the following formula can be introduced:

$$\left[\left(\frac{\mathrm{d}f(t)}{\mathrm{d}t}\right)\Big/\left(\frac{\mathrm{d}l(t)}{\mathrm{d}t}\right)\right]\Big/(S\times L)=E(t), \qquad (2)$$

$S$ is the force bearing area of the specimen, $L$ is the height of the specimen, $l(t)$ is the axial length of the rock samples at the moment of $t$, $f(t)$ is the axial pressure on the rock samples at the moment of $t$.

Substituting eq. (2) into the formula

$$E(t)=af(t)+b \qquad (3)$$

we can get

**Table 1** A comprehensive comparison of sine wave fitting coefficients under different loading frequencies

| Frequency (Hz) | Loading rate(kN/s) | Rate of increase of | Coefficient $a$ | Reduction rate of quadratic | Constant coefficient $b$ |
|---|---|---|---|---|---|

| | | loading rate(%) | | coefficient (%) | |
|---|---|---|---|---|---|
| 0.1 | 19.2 | – | 0.2077 | – | 11.329 |
| 0.2 | 38.4 | 100 | 0.1879 | 9.53 | 11.566 |
| 0.5 | 96 | 150 | 0.1806 | 3.89 | 10.472 |

$$\frac{\mathrm{d}l}{\mathrm{d}t}=\frac{1}{S}\frac{\frac{\mathrm{d}f}{\mathrm{d}t}\times L}{af(t)+b}. \tag{4}$$

By using the above formula, the function expression of $\frac{\mathrm{d}l}{\mathrm{d}t}$ which is not affected by the lag time in loading the sine-wave is generated.

From Figure 4, we can know that load-unload cycle is $T_z$ = 9475 ms; the loading peak rate is

$$\left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)_{\max}=308\times10^{-4}(\mathrm{kN/ms}). \tag{5}$$

Figure 4 accords with sine change law

$$\frac{\mathrm{d}f}{\mathrm{d}t}=\left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)_{\max}\sin\left(\frac{2\pi}{T_z}t\right)=308\times10^{-4}\times\sin\left(\frac{2\pi}{9475}t\right)(\mathrm{kN/ms}), \tag{6}$$

in which, time unit is ms.

Fitting chart of force's change rate is very similar to the original drawing line.

From Figure 4, in the sandstone cyclic loading process when 0.1 Hz sine wave peak is 96 kN,

$$\begin{aligned}f(t)&=\int_0^t\frac{\mathrm{d}f}{\mathrm{d}t}\mathrm{d}t=\int_0^t\left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)_{\max}\sin\left(\frac{2\pi}{T_z}t\right)\mathrm{d}t\\&=\left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)_{\max}\times\frac{T_z}{2\pi}\times-\cos\left(\frac{2\pi}{T_z}t\right)\Bigg|_0^t\\&=\left(\frac{df}{dt}\right)_{\max}\times\frac{T_z}{2\pi}\times[1-\cos\left(\frac{2\pi}{T_z}t\right)]\\&=308\times10^{-4}\times\frac{9475}{2\pi}\times[1-\cos\left(\frac{2\pi}{9475}t\right)],\end{aligned} \tag{7}$$

besides, $L$=100 mm, $S=\pi R^2$ = 3.1415×(54/2)$^2$ = 2290 (mm$^2$).

From the preceding analysis, we can obtain

$$a=0.2067\ (10^6\,\mathrm{N/m^2}),\quad b=11.39\,(\mathrm{GPa}).$$

Then they are substituted into the following formula:

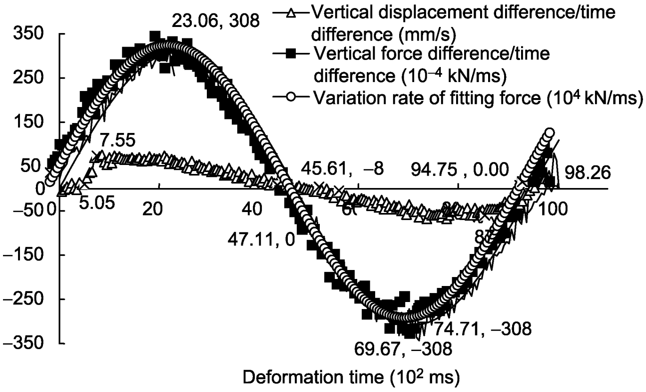


**Figure 4** Fitting chart of force's change rate with time when sine wave frequency is 0.1Hz.

$$\frac{\mathrm{d}l}{\mathrm{d}t}=\frac{1}{S}\frac{L\times\frac{\mathrm{d}f}{\mathrm{d}t}}{af(t)+b}=\frac{1}{2290}$$

$$\times\frac{100\times308\times10^{-4}\times\sin\left(\frac{2\pi}{9475}t\right)}{0.2067\times308\times10^{-4}\times\frac{9475}{2\pi}\times[1-\cos\left(\frac{2\pi}{9475}t\right)]+11.39},$$

$$\times10^6(\mathrm{mm/s}), \tag{8}$$

in which, the time unit is ms.

The above formula is the cyclic loading rate calculation formula in the sandstone cyclic loading process when 0.1 Hz sine wave peak is 96 kN.

### 5.2 Comparison with the measured values and the explanation of "peak shifting phenomenon"

In Figure 5, the calculated value is in perfect agreement with the measured value, which shows that the assumptions and derivation of computational model are reasonable. As in formula (4), the deformation rate is codetermined by $\frac{\mathrm{d}f}{\mathrm{d}t}$ and $f(t)$ according to a certain function, which well explains the "peak shifting phenomenon" between the curve peaks of $\frac{\mathrm{d}l}{\mathrm{d}t}$ and $\frac{\mathrm{d}f}{\mathrm{d}t}$ with time changing. The micro-mechanism of "peak shifting phenomenon" is based on formula (3), that is, as the load increases, the rock samples have been gradually compacted, the further deformed margin decreases, under the same increase of the load, the deformation gradually decreases, showing that the deformation resistance and the apparent tangent modulus increase. If we use the general approximation theory and ignore the impact of rock sample compaction on the apparent tangent modulus of rock mass, $\frac{\mathrm{d}l}{\mathrm{d}t}$ will only be determined by $\frac{\mathrm{d}f}{\mathrm{d}t}$. The variation between the two shows a linear relation, and the peaks will occur at the same time.

The aim of exploring the "peak shifting phenomenon" is to prove that the extent of the rock sample meso-pressure effects on the elastic modulus values can be reflected by $f(t)$ through testing, which leads to a tremendous impact on $\frac{\mathrm{d}l}{\mathrm{d}t}$. It is also to point out the nonlinear change relationship between $\frac{\mathrm{d}l}{\mathrm{d}t}$ and $\frac{\mathrm{d}f}{\mathrm{d}t}$ and to demonstrate that the development of both $\frac{\mathrm{d}l}{\mathrm{d}t}$ and $\frac{\mathrm{d}f}{\mathrm{d}t}$ is non-synchronized and the times of two extremes have a greater difference. The importance of simulating the "peak shifting phenomenon" accurately is as follows: the relationship between the rock sample's axial energy absorbed and the time can be accurately calculated, and a new theoretical basis for the corresponding analysis of the mechanics and theoretical research can be provided when rock samples are under cyclic loading in the earthquake.

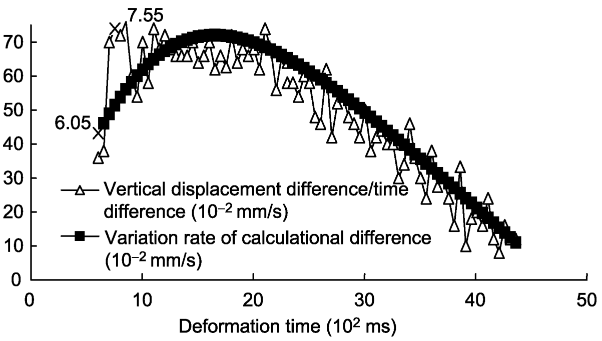


**Figure 5** Comparison chart of the calculated and measured displacement rate of sandstone when sine wave frequency is 0.1 Hz.

In Figure 6, stress-strain diagrams are in a good agreement, aberrations in the middle are caused by certain changes of the initial displacement at the secondary loading stage. If the initial value agrees with the measured value after the secondary loading, the comparison chart is shown in Figure 7.

In Figure 7, stress-strain diagram from the corrected forecast is in a very good agreement with the measured one, illustrating the rationality of the above analysis and calculation. The derivation formulas of the segment without hysteretic effect are employed to calculate the whole section,

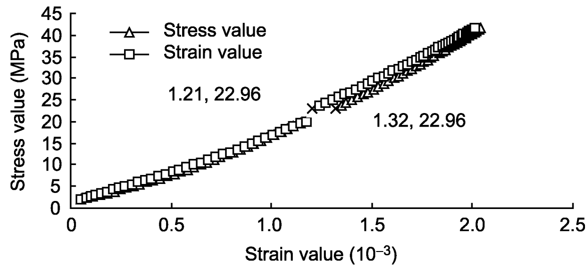


**Figure 6** Comparison between the stress-strain measured value and calculated value in loading segment without considering the influence of lag time.

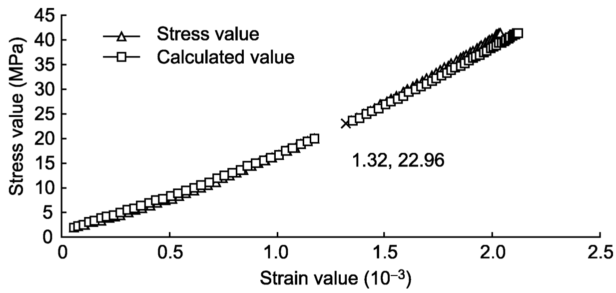


**Figure 7** Comparison of the measured stress-strain diagram and calculated values of loading segment that are not influenced by lag time segments (corrected initial values).

and the calculated values with the measured ones are also compared (see Figure 8).

In Figure 8, the comparison shows that the numerical calculation accords with the experiment result very well in the whole loading section except the initial displacement rate which is a little different.

As Figure 9 shows, the forecasted stress-strain diagram is obviously different from the experiment result in the whole loading section.

### 5.3 Formulas correction of the loading lag section and the compactions with the calculated values

*5.3.1 Presumption and hypothesis of the established formula and the corrected formula.*

The comparison shows if the displacement of the starting point of the uniform acceleration is required to be equal to the measured value, the forecasted value of calculation time 0 should be negative, and the agreement of the curves is poor. Through the above experimental analysis, this segment is a lag segment of displacement rate relative to loading rate. The following is the correction of the calculating curve with the influence of lag time taken into account.

The starting value and the inferential reasoning premise in delay section (similar to the premise in the triangular wave analysis):

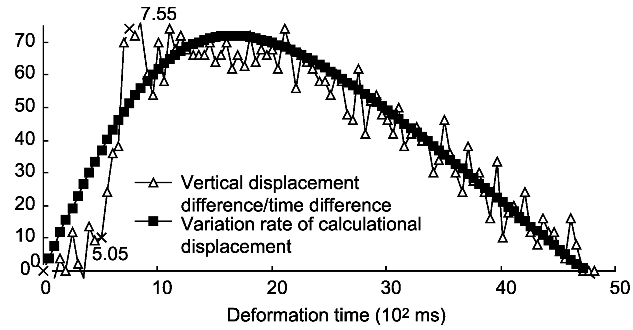


**Figure 8** Comparison of the calculated and measured displacement rate of sandstone during the sine wave loading segment of frequency 0.1 Hz.

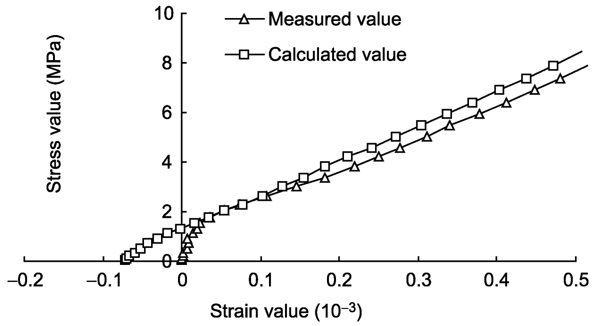


**Figure 9** Comparison of the measured stress-strain diagram and calculated values of loading segment without considering the influence from lag time.

(i) When t=0, $\frac{\mathrm{d}f}{\mathrm{d}t}=0$, $\frac{\mathrm{d}l}{\mathrm{d}t}=0$, $f$=0, $\frac{\mathrm{d}^2 l}{(\mathrm{d}t)^2}=0$.

(ii) In the delay time section, the rate of displacement has not been developed, $\frac{\mathrm{d}l}{\mathrm{d}t}=0$.

(iii) After the delay time ends, the rate of displacement rapidly expands, and the displacement reaches the peak value, then it reaches the value in actual uniform speed section. So we must determine the delay time $t_z$, the time segment $t_f$ to reach the end point of the hysteretic time influence when the hysteretic time ends, and the initial value $l_y$ of the displacement in the section of no-delay time influence.

(iv) According to the measured data, we suppose that the speed of the displacement grows evenly in the time section $t_f$. So: $\int_0^{t_f}\frac{\mathrm{d}l}{\mathrm{d}t}\mathrm{d}t=l_y$, and $\frac{\mathrm{d}^2 l}{(\mathrm{d}t)^2}=c_1$, $c_1$ is the constant, then $\frac{1}{2}c_1 t_f^2=l_y$, $c_1=2\frac{l_y}{t_f^2}$.

(v) In addition, as Table 2 shows that 351 ms is the delay time, namely $t_z$=351 ms. It takes 404 ms to reach the starting value point of no-delay time influence section at the latter stress speed. The measured data of this point are listed in Table 2.

### 5.3.2 Comparison of displacement rate

Considering the contrast between the predicted value and the actual value of the delay time, we give the model calculation example that the delay time is added into the preliminary stage of the loading segment:

In the sine wave cyclic loading process (0.1 Hz, the peak value of sandstone is 96 kN), $c_1=2\frac{l_y}{t_f^2}=1.7768\times10^{-7}$ $(\mathrm{mm/ms^2})$, when $t_z\leqslant t\leqslant t_f$, $\frac{\mathrm{d}l}{\mathrm{d}t}=c_1(t-t_z)$ the forecast curve of the displacement rate is got (Figure 10).

Figure 10 shows that the preliminary stage considering the delay time fits very well with the actual value, and it explains that the hysteretic effect has obviously affected the initial period development of the strain's speed.

### 5.3.3 Stress-strain chart comparison (Figure 11)

The stress-strain curve after the delay time tallies well in the section influenced by the delay time. Except that the forecast speed peak value is slightly high, this computation model's computed result tallies well with the actual result. This also shows the reason why the peak value of load rate appears. In the initial loading period, the displacement rate development relatively lags behind the loading rate development, but they will achieve the synchronization in the starting point of a certain start point after the lagging effect influences segment. And because of the vanishing of lag, the stress and the displacement will also conform to the corresponding rule of the later period, namely this beginning displacement $l_y$ is determined. It is also the critical displacement between the load initial period and load later period. Its characteristics are: after this displacement, the deformation rate in the load section will not be influenced by the delay time, and it completely tallies with the previous derivation formula of uniform speed section. The displacement rate reaches peak value at 50 ms before this displacement. The law of the displacement rate before this point is entirely different from that after this point.

If the displacement gradually achieves $l_y$ at an increasing

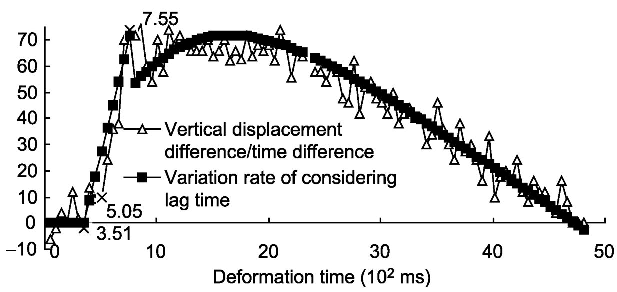


**Figure 10** The contrast chart between the computation and actual values of the displacement rate in sandstone's sine wave (0.1Hz) (considering the delay time).

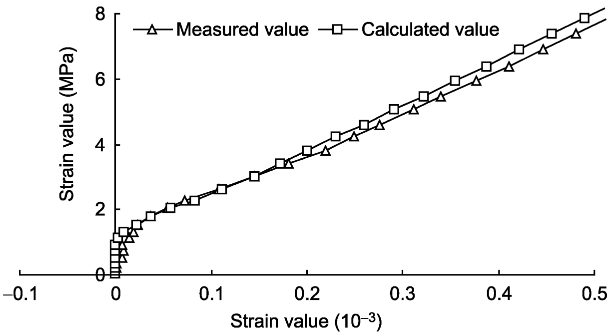


**Figure 11** Contrast chart between the computation and actual values of stress-strain curve in sandstone's sine wave (0.1 Hz) (considering the delay time).

**Table 2** Initial point data statistics

| Deformation time($10^2$ ms) | Time (ms) | Vertical force (kN) | Vertical deformation (mm) | Transverse deformation (mm) | Apparent tangent modulus (GPa) | Vertical force difference (kN) | Vertical displacement difference/time difference ($10^{-3}$ mm/s) | Vertical force difference/time difference (kN/ms) | Strain value ($10^{-3}$) | Stress value (MPa) |
|---|---|---|---|---|---|---|---|---|---|---|
| 7.55 | 190029 | 6.92 | 0.0735 | 0.0145 | 11.3301074 | 0.96 | 74 | 0.0192 | 0.145 | 3.021834 |

rate in a short-time section $t_f$ that removes the delay time and then the displacement rate returns to be continual with speed change in the later period, definitely it will experience a peak value of speed development.

*5.3.4 Comparison of apparent module chart (Figure 12)*

The peak value of the apparent tangent modulus also appears at the beginning of the load section when the delay time is considered. In the delay time section, because suppose that strain value is 0, the apparent tangent modulus tends infinity. The predicted value in the first 351ms has not been listed. The change in the entire later period tallies very well, and it explains that the calculation model in which the delay time is considered can well describe the dynamic change of rock mass's mechanics parameter in the sine wave load section. The change in the later period does not fit, this is mainly because the fluctuation of the stress speed is around 0.

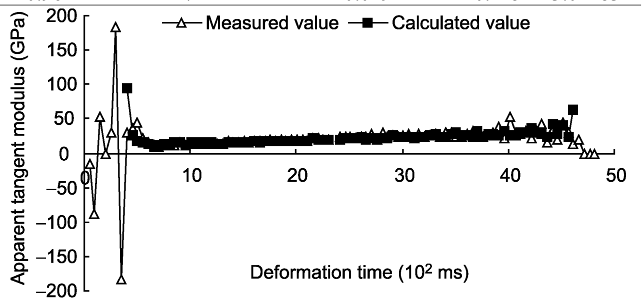


**Figure 12** Relationship between apparent tangent modulus (GPa) and time (considering the delay time in load initial period).

## 5.4 Simulation structure of different frequencies

According to the above simulation formula and the step, the simulation to the 0.2 Hz and 0.5 Hz sine waves is analyzed, with the influence of the delay time unconsidered (see Figure 13 and 14).

The simulation of deformation rate indicates that by applying the above-mentioned simulate formulas and steps, the calculated values coincide well with the measured val-

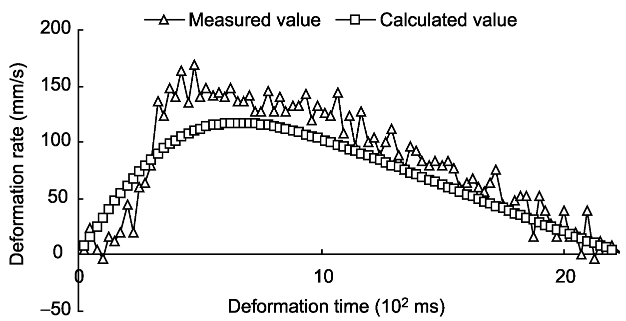


**Figure 13** Relationship between the vertical deformation rate and the time of sandstone under sine wave load of 96 kN amplitude value with the frequency of 0.2 Hz.

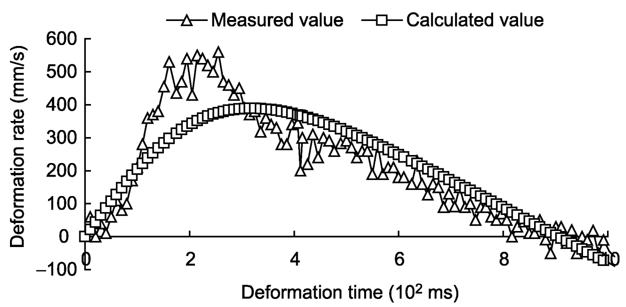

**Figure 14** Relationship between the vertical deformation rate and the time of sandstone under sine wave load of 96 kN amplitude value with the frequency of 0.5 Hz.

ues. The loading and unloading processes of the rock sample can also be accurately simulated when the deformation rate increases gradually to the peak then decreases gradually even without considering the lag time. So we conclude that the above simulation formulas are suitable for the simulation of the deformation rate of sandstone under the sine wave load of 96 kN amplitude value and frequency of 0.1—0.5 Hz.

## 6 Conclusions

(1) In order to reduce the data variance caused by individual differences of different samples, the paper presents a new experimental method of loading and unloading the axial force with different frequencies and different waves of a certain load amplitude to the same rock sample without causing damages to it.

(2) The lag time segments are defined and fractionized into segment I and segment II, and criterions for segmentation, definitions of relevant parameters and empirical analysis are provided. The peak value dislocation of the displacement variation rate and the loading rate of force in the course of sinusoidal loading is then defined as peak dislocation.

(3) It also defines the apparent tangent modulus. The test data are used to prove that the linear relation $E(t) = af(t) + b$ between the apparent tangent modulus and the vertical force exists all the time in the frequency of 0.1, 0.2, 0.5 Hz sinusoidal load segments.

(4) The calculating formula of the deformation rate in the non-lag time is deduced. It is indicated that the loading rate df/dt and instant load $f(t)$ codetermine the deformation rate, which well explains the peak dislocation of the time-variable curve peak value of $dl/dt$ and deformation rate of $df/dt$. The comparison and analysis of the calculated values and measured values demonstrate the influences of lag time segments on the deformation rate and the stress-strain diagram, thus a derived model based on lag time segment has been built up.

(5) Comparison between the calculated values and measured values shows that the above formula can be a more accurate simulation of the deformation rate of sandstone with the load amplitude being 96 kN and the frequency ranging from 0.1 Hz to 0.5 Hz.

Due to the limitation of the space, the simulation and prediction of the deformation rate of the unloading section will be described in another paper.

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