A modified approach for determination of nonlinear properties in seismic response analyses for 200 m high core rock-fill dams

Hongjun Li, Shichun Chi, Gao Lin, and Hong Zhong

Abstract: Equivalent linear analyses are widely used for estimation of site response and safety status of rock-fill dams subjected to strong earthquakes. However, the average normalized curves incorporated in the equivalent-linear iterative process cannot precisely depict the variations of dynamic parameters with shearing strain for one type of soil material under a wide range of confining pressures. Thus, a modified approach for the determination of nonlinear properties for soil elements confined under a broad range of effective pressures in site response analyses for high rock-fill dams (>200 m) is proposed. In this approach, the normalized confining-pressure-interpolating curves (CPI curves) of each soil element under different effective stress are obtained by linearly interpolating or extrapolating by its in situ stress. By comparing the results obtained by equivalent linear analyses incorporating the average curves and the CPI curves, respectively, the impact of utilizing the recommended curves when determining nonlinear soil properties on site response analyses of high rock-fill dams (>200 m) is discussed. It is shown that the refinement in the determination of nonlinear properties during site response can be utilized in the near future by incorporating the results of this study in practice.

Key words: confining pressure, average curve, CPI curve, nonlinear soil properties, site response, high rock-fill dam.

Résumé : Des analyses linéaires équivalentes sont communément utilisées pour estimer la réaction du site et l'état de sécurité des barrages en enrochements soumis à de forts séismes. Cependant, les courbes moyennes normalisées incorporées dans le processus itératif équivalent-linéaire ne peut pas décrire précisément les variations des paramètres dynamiques avec la déformation en cisaillement pour un type de matériau de sol sous une large plage de pressions de confinement. Alors, on propose une approche modifiée pour la détermination des propriétés non linéaires pour des éléments de sol confinés sous une large plage de pressions effectives dans des analyses de réaction de site pour les barrages élevés en enrochements (>200 m). Dans cette approche, les courbes normalisées de confinement-pression-interpolation (courbes CPI) de chaque élément de sol sous différente contrainte effective sont obtenues en interpolant ou extrapolant par sa contrainte in situ. En comparant les résultats obtenus par des analyses linéaires équivalentes incorporant les courbes moyennes et les courbes CPI respectivement, on discute l'impact de l'utilisation des courbes recommandées pour déterminer les propriétés du sol non linéaire dans les analyses de réaction du site de barrage élevé en enrochement (>200 m). On montre que le raffinement dans la détermination des propriétés non linéaires durant la réaction du site peut être utilisé très prochainement en incorporant les résultats de cette étude dans l'état de la pratique.

Mots-clés : pression de confinement, courbe moyenne, courbe CPI, propriétés de sol non linéaires, réaction de site, barrage élevé en enrochements.

[Traduit par la Rédaction]

Introduction

A number of rock-fill dams with heights greater than 200 m are being, or will be, built in the southwest area of China, an area with high seismic risk. This has made the issue of dam safety against strong earthquake shocks one of great concern. Various analytical approaches ranging from shear beam analyses to fully nonlinear analyses are now available for the prediction of seismic response. These approaches simulate the mechanics of the seismic response problem with different levels of precision and require different information on material behavior. The equivalent linear analytical approach is considered a most useful tool that can represent important physical mechanisms of seismic response problems using material information that can be obtained easily and economically.

As is well known, the accuracy of analytical methods for dynamic problems depends to a large extent on the cyclic stress–strain characteristics of soil in shear, including small-strain shear modulus (G_{max}) and the relationships between the dynamic properties and cyclic shearing strain (G/G_{max}~γ, ξ~ γ, where G is the secant dynamic shear modulus, γ is the shearing strain, and ξ is the damping ratio). Over the past three decades, numerous studies have been conducted regarding dynamic soil properties and the parameters affecting them. Many researchers have noticed

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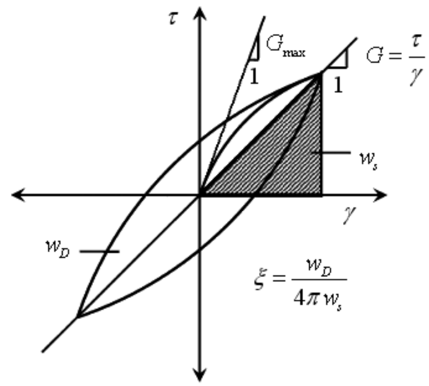
the effect of confining pressure on the nonlinear soil properties and proposed various nonlinear empirical confining-pressure-dependent curves for use in earthquake analyses. This discussion will assess the shortcomings of the existing empirical confining-pressure-dependent curves proposed by previous researchers. In addition, the strengths and limitations of the refined approach developed in this paper will be easily identified in terms of the distribution of dynamic parameters, seismic response results, and indexes of safety evaluations.

Nonlinear dynamic soil properties

The basic nonlinear stress–strain relationship of soils, as schematically depicted in Fig. 1, is a prime factor to be considered in the dynamic analyses of earth structures. There are three fundamental parameters: G_{max}, G, and ξ. Numerous analytical methods use the normalized modulus (G/G_{max}) reduction and material damping (ξ) curves, which vary with amplitude of shearing strain (γ), to determine the nonlinear properties of soil material in dynamic analyses. The first comprehensive study in which the parameters that control nonlinear soil behavior were identified was the study by Hardin and Drnevich (1972*a*, 1972*b*). The empirical equations and curves proposed by Hardin and Drnevich (1972*a*, 1972*b*) account for the effect of confining pressure mainly through adjustment of the reference strain. The complexity of determining the normalized modulus reduction and material damping curves limited its application in practice.

Many other researchers have been influenced by the Hardin and Drnevich (1972*a*, 1972*b*) work and have attempted to refine, improve, and generalize their results. The effect of mean effective confining pressure on normalized modulus reduction curves is discussed based on resonant column and torsional shear tests using hollow specimens confined at low confining pressure ($\sigma_0' < 200$ kPa) by Iwasaki et al. (1978). Seed et al. (1986) provided a well-established benchmark for dynamic properties of sandy soilsthat was based on numerous early laboratory results primarily for confining pressures corresponding to the upper 10 m of a soil column. Vucetic and Dobry (1991) developed a set of confining pressure – independent design curves for a wide range of plasticity indexes based on laboratory data at relatively low confining pressures. Ishibashi and Zhang (1993) showed that G/G_{max} generally decreased less with increasing shearing strain as confining pressure increased and offered the empirical equations for $G/G_{max}\sim\gamma$ and $\xi\sim\gamma$. Pyke (1993) developed a set of confining-pressure-dependent curves based on a combination of theory and an extensive new testing program by Stokoe (1993) on natural samples obtained over relatively wide ranges of confining pressures. Darendelli (2001) developed a simple geoindex model for selection of dynamic properties in seismic response analyses and found that utilizing a family of confining-pressure-dependent curves would result in larger intensity ground motions than those predicted with average generic curves obtained by Seed et al. (1986). In these other studies, "average" normalized modulus reduction and material damping curves are widely accepted and utilized in practice. Although the importance of effective confining pressure on the variation of normalized shear modulus and material damping ratio with shearing strain is addressed in previously mentioned studies, it is important to note that the average empirical curves and equations proposed by the previous researchers are based on data collected at a relatively low confining pressure, and these curves and empirical equations do not quantify the effect of confining pressure on nonlinear soil behavior as soil specimens confined at a wider range of confining pressure.

Fig. 1. Nonlinear cyclic stress–strain relationship of soils.



As part of the project for the 240 m ChangHe core rock-fill dam, the dataset from the China Institute of Water Resource and Hydropower Research (IWHR) provided an excellent opportunity to develop a refined approach by further considering the effect of confining pressure on the shape of normalized dynamic parameter curves and seismic response results. In this modified approach, each soil element is assigned a set of unique and time-dependent nonlinear soil property curves according to its current effective stress, in contrast to the previous methods, which synthesized a set of average curves for one type of soil material regardless of the obvious differences among soil elements.

Correlations between G_{max}, G/G_{max} versus γ and effective confining pressure

According to laboratory data, for small strains, that is, within the range of about 1×10^{-6}, the area of the loop is almost nonexistent, and the relationship between shear stress and shear strain is nearly a straight line. This means that soil may be handled as a linearly elastic material, where the slope of the stress–strain line is equal to the maximum shear modulus (G_{max}). The G_{max} can be obtained from in situ measurement and laboratory tests, such as the resonant column test and torsional shear tests. Hardin and Drnevich (1972*b*) developed the following empirical equation based on laboratory and in situ measurements to evaluate the maximum shear modulus for rock-fill materials in seismic response analyses.

$$[1] \qquad G_{max} = K\mathrm{Pa}\left(\frac{\sigma_0'}{\mathrm{Pa}}\right)^n$$

To further refine the influence of the effective confining pressure on the variation of normalized shear modulus with shearing strain, the curves of four major filling materials from the ChangHe dam confined at a wide range of confining pressures are presented in Fig. 2. The data show that (*i*) the normalized modulus reduction data for four types of soils become increasingly linear as confining pressure in-

Fig. 2. Relationships between G/G_{max} and γ for four types of soils under a wide range of confining pressures: (*a*) filter material, (*b*) core material, (*c*) shell material, (*d*) transitional material.

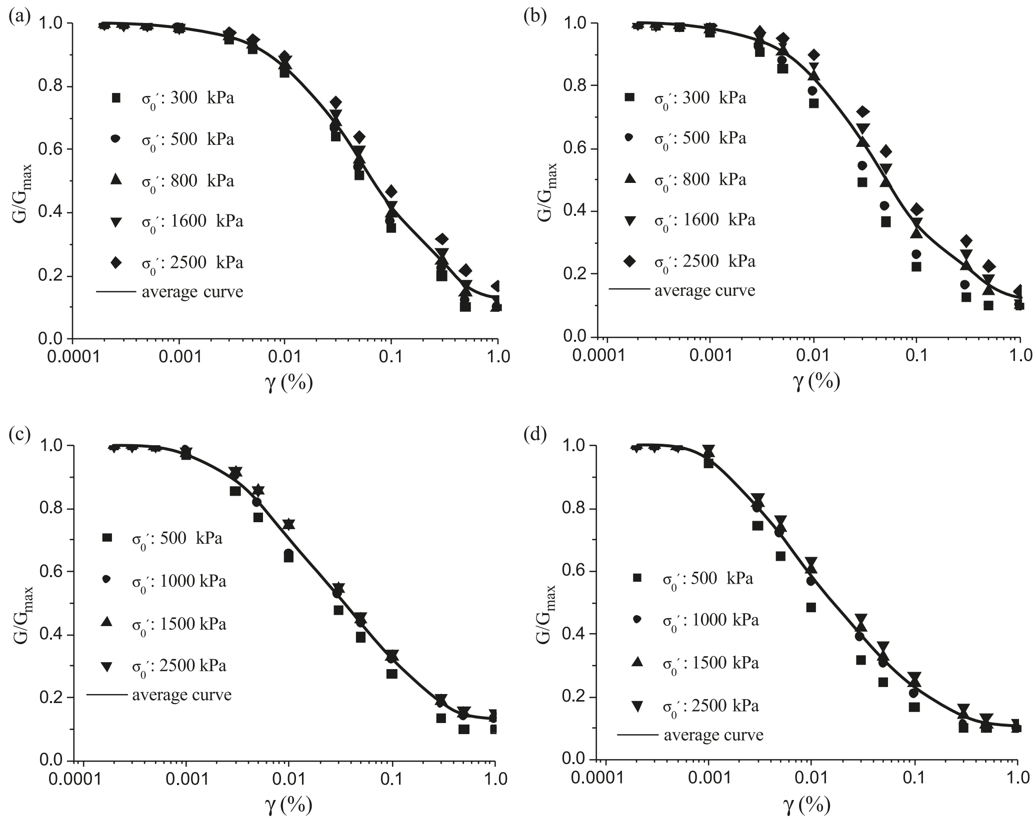


Fig. 3. Correlations between G/G_{max} and effective confining pressure for four types of soils: (*a*) core material, (*b*) filter material, (*c*) shell material, (*d*) transitional material.

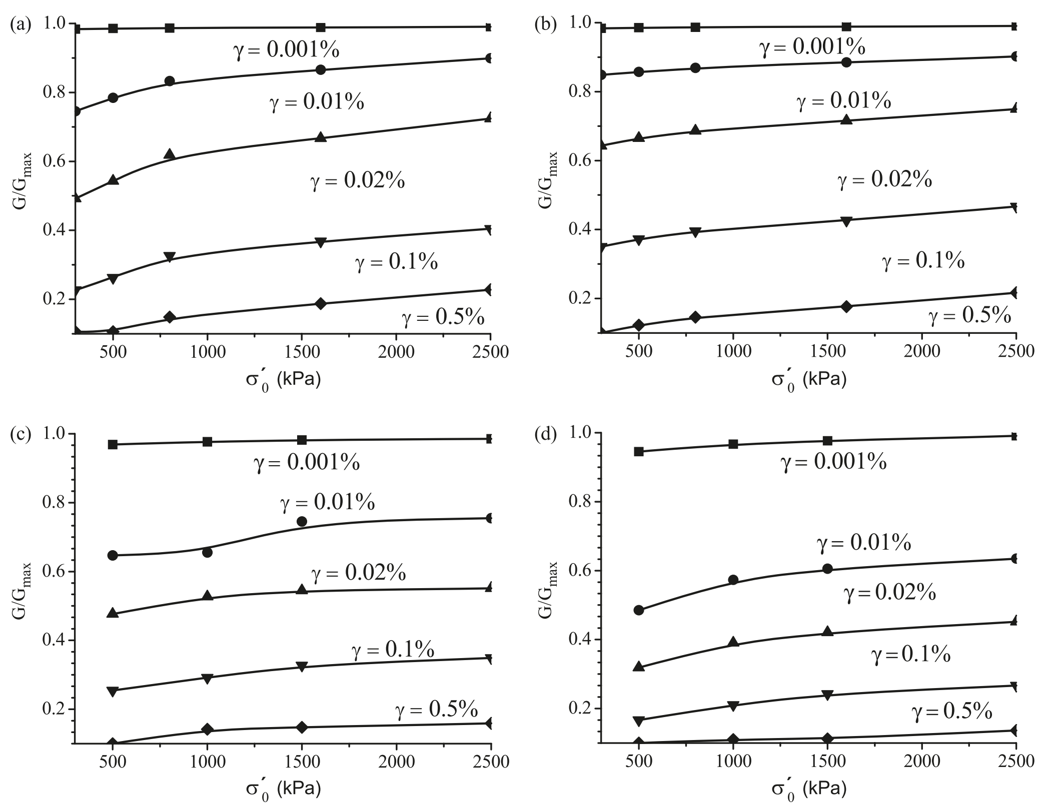
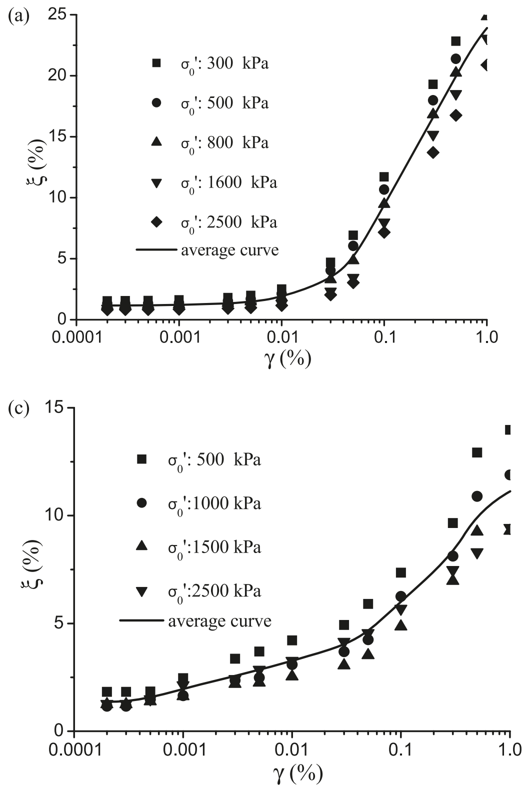


Fig. 4. Relationships between ξ and γ for four types of soils under a wide range of confining pressures: (*a*) filter material, (*b*) core material, (*c*) shell material, (*d*) transitional material.



creases; (*ii*) the effect of the confining pressure on G/G_{max} versus γ can be neglected as the amplitude of shearing strain is below 1×10^{-5}; (*iii*) the data are more scattered under a wide range of confining pressures when the amplitude of shearing strain is in the range of 1×10^{-5} to 1×10^{-3}; and (*iv*) these average normalized modulus reduction curves do not fully characterize the relationship between normalized modulus and shearing strain under a wide range of confining pressures as the shearing strain reaches moderate amplitude.

To further quantify the influence of confining pressure on the normalized modulus, the relationship between confining pressure (σ'_0 = 300–2500 kPa) and the normalized modulus for some given amplitude of shearing strains for the four major filling materials are presented in Fig. 3. Considerable increase in the normalized modulus is observed when the given shearing strain is above 0.01%.

Correlations between ξ–γ and effective confining pressure

The relationships between the damping ratio ξ and the amplitude of shearing strain γ at a wider range of confining pressure is depicted in Fig. 4. The following observations can be made from these charts: (*i*) the material damping data for four types of soils become increasingly linear as confining pressure increases; (*ii*) the influence of confining pressure on ξ versus γ is negligible at $\gamma < 1 \times 10^{-5}$; (*iii*) the deviation of laboratory data under a wide range of confining pressures (σ'_0 = 300–2500 kPa) from the average material damping curves is increasingly evident when the shearing strain is above 0.001%.

To further quantify the influence of confining pressure on the damping ratio, the relationships between confining pressure and material damping for some given amplitude of shearing strains for the four filling materials are presented in Fig. 5. Considerable decrease in damping ratio is observed when the given shearing strain is above 0.001%.

Analytical procedure

Equivalent linear analyses are often performed to estimate the dynamic response of high core rock-fill dams; nonlinear properties can be obtained through a series of linear iterations. A set of initial values for the modulus and damping should be assumed at the beginning of the time-history analyses. These are updated using one model of characterizing the relationship between dynamic parameters and shearing strain in the subsequent time steps, where equivalent linear analyses are performed in each step. Rayleigh's concept of proportional damping is employed to represent the hysteretic damping of soil. The Wilson-θ algorithm is utilized in the solution of the dynamic equations in the time marching process.

In the traditional equivalent linear analyses, the average dynamic parameter curves are usually employed for the determination of nonlinear properties during the linear iterative process. However, for 200 m high core rock-fill dams like those located in the southwest area of China, the effective

Fig. 5. Correlations between ξ and effective confining pressure for four types of soils: (*a*) core material, (*b*) filter material, (*c*) shell material, (*d*) transitional material.

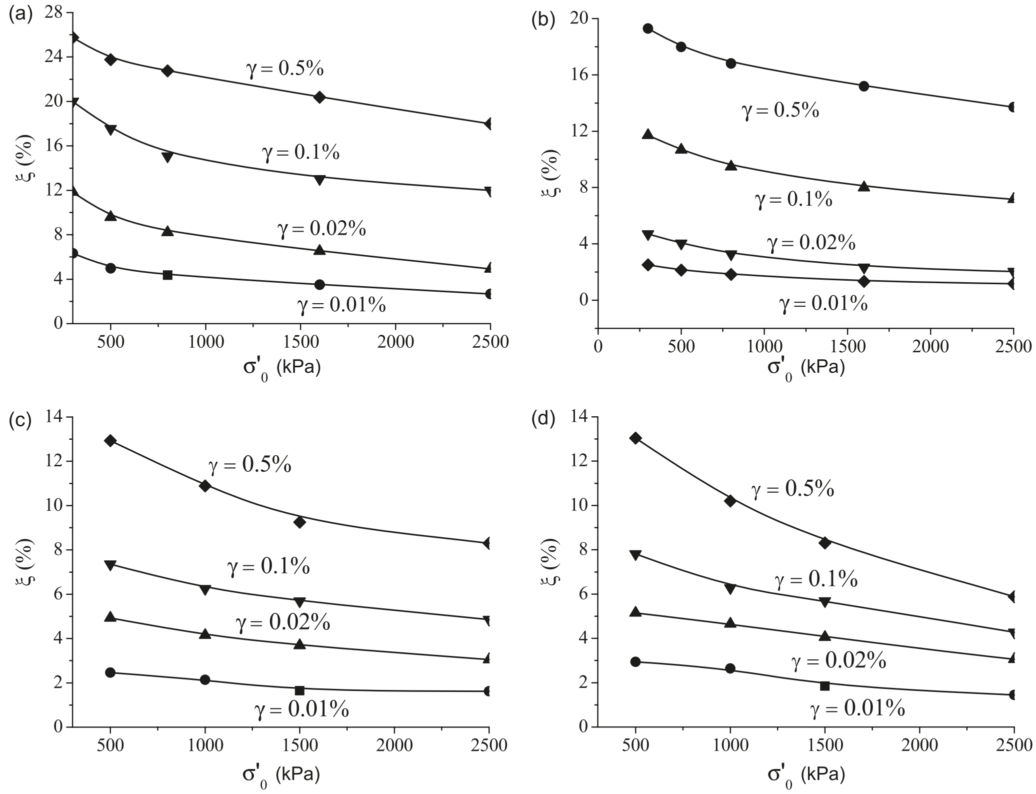


Fig. 6. Finite element discretization of the ChangHe dam.

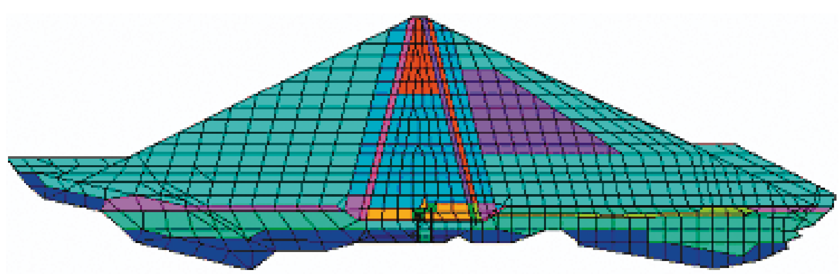
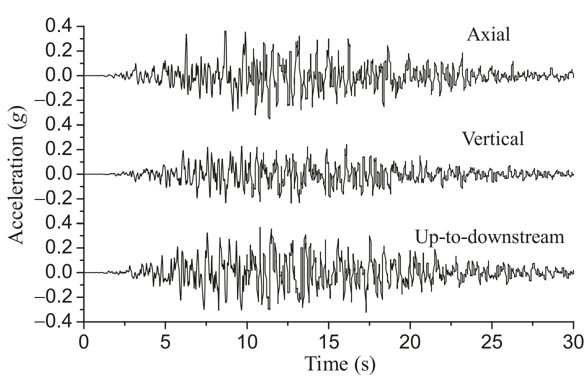


Fig. 7. Accelerograms used in the analyses.



stress of soil elements in the dam may range from zero to several thousand kilopascals, which would have a significant influence on the distribution of the dynamic properties.

As shown in Figs. 2 and 4, the laboratory data under a wide range of confining pressures for the four types of filling materials show two common trends: (*i*) the normalized modulus reduction and material damping curves tend to be increasingly linear as confining pressure increases and (*ii*) the average dynamic parameter curves could not get a good fit to the data measured in the laboratory under a wide range of confining pressures. It is worthwhile developing an improved approach that can bridge the gap between the traditional model and the "competent" model.

In the equivalent linear analyses the refined confining-pressure-interpolating (CPI) curves are incorporated into each iterative process in place of the average normalized dynamic parameter curves. One of the major strengths of the modified model is due to the fact that each soil element in the rock-fill dam can have a set of unique and time-dependent nonlinear dynamic parameter curves that can be obtained by linear interpolation or extrapolation of the laboratory data at a wide range of confining pressures according to its current mean effective stress. Compared with the tra-

Fig. 8. Distributions of nonlinear soil properties at the upstream surface for the four cases A, B, C, and D referred to in Table 1: (*a*) normalized shear modulus, (*b*) material damping.

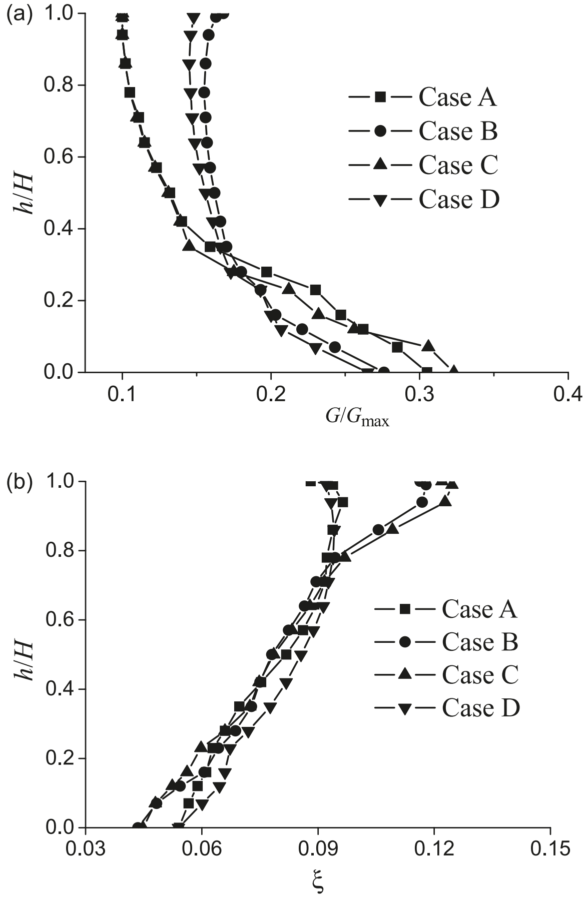
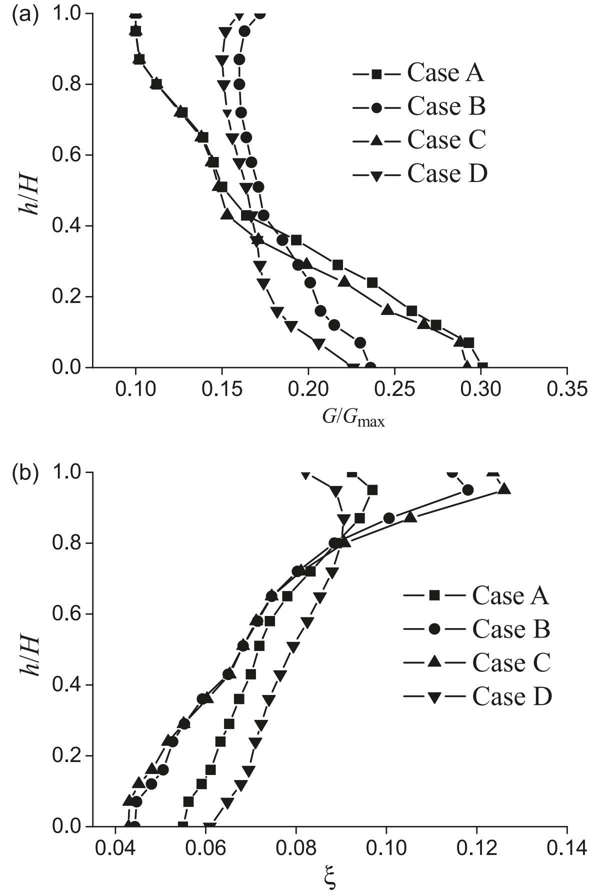


Fig. 9. Distributions of nonlinear soil properties at the downstream surface for the four cases A, B, C, and D referred to in Table 1: (*a*) normalized shear modulus, (*b*) material damping.



ditional model that assigns a set of average nonlinear dynamic parameter curves for one type of soil material, the refined model can characterize the relationship between nonlinear soil properties and shearing strain precisely for every soil element rather than for only one type of soil material. It does, however, require more storage space and calculation time.

Seismic response analyses of a 240 m high core rock-fill dam

The charts in Figs. 3 and 5, which show the influence of the effective confining pressure on $G/G_{\max}$ and ξ for some given shearing strains, can be meaningful to seismic response and safety evaluation. As far as the high core rock-fill dams (>200 m) are concerned, the difference in effective confining pressure within the dam would cause or enhance the heterogeneity of dynamic properties due to their dependency on confining pressure. The following is a numerical example for the ChangHe dam, a clay-core rock-fill dam being constructed in southwestern China. Its crest is 16 m wide and 240 m high, and the maximum cross section is plotted in Fig. 6. The dam is mainly composed of core, filter, shell, and transitional material, and the relationships between dynamic parameters and shearing strain for these filling materials are presented in Figs. 2 and 4.

Ground motion and design cases

The design ground motion was selected based on in situ investigations and is plotted in Fig. 7. Its amplitude is $0.366g$ in both upstream to downstream and axial directions and $0.241g$ in the vertical direction. In the time-marching process of equivalent linear analyses, the duration of the earthquake excitation (30 s) is divided into 15 periods.

In our research we considered the four computation cases listed in Table 1. Site response analyses for the 240 m high rock-fill dam for these four cases were carried out to evaluate the impact of modeling confining-pressure-dependent nonlinear soil properties on the distribution of dynamic parameters, absolute acceleration, permanent deformation, and liquefaction.

Distribution of dynamic parameters

Corresponding to the four cases listed in Table 1, the maximum nonlinear soil properties along the height of the dam in the time marching process are shown in Figs. 8 and 9. As can be seen, the distributions of nonlinear dynamic parameters along the height of dam for the four cases exhibit obvious discrepancies at both the upstream and downstream faces. The ratios of $G/G_{\max}$ and ξ predicted using the average normalized nonlinear soil property curves to those predicted with the CPI model are 73% and 133%, respectively,

Table 1. Cases incorporating different models of obtaining nonlinear dynamic properties in the ChangHe dam project.

Case	G/G_{max}~γ	ξ~γ
A	CPI curve	Average curve
B	Average curve	CPI curve
C	CPI curve	CPI curve
D	Average curve	Average curve

Note: G/G_{max}, maximum small-strain shear modulus; γ, shearing strain; ξ, damping ratio; CPI, confining-pressure-interpolating.

at the bottom of the dam, and they are 160% and 61%, respectively, at the top of the dam. At the downstream face, G/G_{max} decreases while ξ increases along the height of the dam for the four cases, but the degree of variation for the four cases is significantly different. Consequently, there exist intersections among the curves, to be specific, at h/H = 0.40–0.45 for the G/G_{max} curves and h/H = 0.8 for the ξ curves in Fig. 8. The dynamic properties at the upstream face and the downstream face are slightly different, owing to the pore-water pressures, which change the distribution of the effective stress along the height of dam.

Absolute acceleration

In Fig. 10 and Table 2, some comparisons of the predicted ground motions in terms of the maximum absolute acceleration and its distribution along the height of the dam for the four cases are presented. The maximum absolute accelerations listed in Table 2 reveal that the traditional method as employed in case D, which incorporates the average normalized nonlinear dynamic parameter curves into linear iterative analyses, generates conservative results for the evaluation of a high rock-fill dam. The ratios of the maximum absolute acceleration predicted in case D to those predicted in cases A, B, and C are 105.6%, 118.1%, and 112.7%, respectively, in the upstream–downstream direction; 92.9%, 109.3%, and 97.7%, respectively, in the vertical direction; and 120.0%, 108.2%, and 122.8%, respectively, in the axial direction. In addition, the distributions of absolute acceleration in the upstream–downstream direction, vertical acceleration, and axial acceleration along the axis of the ChangHe dam for the four computation cases are shown in Fig. 10. It can be seen in Figs. 10*a* and 10*c* that response acceleration at the dam crest in case D is greater than those in other cases. For an overall comparison of the acceleration obtained from the four computation cases, considerable difference is observed for h/H greater than 0.5. Additionally, the whiplash effects for cases A, B, and C incorporating the CPI curves decrease to some degree compared with case D incorporating the average curves.

Permanent deformation

As an index for the safety evaluation of high rock-fill dams, the accuracy of the vertical settlement is very important. The equivalent linear analysis is often employed to get the dynamic stress of soil elements in the dam, which is then used to predict the strain potential and permanent deformation. Hence, the four cases shown in Table 1 are analyzed to investigate the influence of the dependency of dynamic properties on confining pressure on permanent deformations. The maximum permanent deformations in three directions are listed in Table 3. The ratios of vertical settlement predicted in case D to those predicted in cases A, B, and C are 125.2%, 113.6%, and 118.5%, respectively.

Table 2. Results of the maximum absolute acceleration of the ChangHe dam.

Case*	Upstream to downstream (g)	Vertical (g)	Axial (g)
A	0.927 (2.53)	0.685 (2.81)	0.726 (1.98)
B	0.829 (2.26)	0.583 (2.39)	0.805 (2.20)
C	0.869 (2.37)	0.687 (2.82)	0.709 (1.94)
D	0.979 (2.67)	0.637 (2.61)	0.871 (2.38)

Note: The number in the bracket is the amplification factor obtained by the maximum acceleration divided by the amplitude of the input ground motion in each direction.

*The meaning of A, B, C, and D is defined in Table 1.

Table 3. Results of the maximum permanent deformation of the ChangHe dam.

Case*	Upstream to downstream (cm)	Vertical (cm)	Axial (cm)
A	17.1	90.6	19.1
B	24.1	99.9	25.45
C	22.1	95.7	23.14
D	24.95	113.5	29.89

*The meaning of A, B, C, and D is defined in Table 1

It is implied in Table 3 that the dependency of nonlinear soil property curves on effective confining pressure can greatly influence the vertical settlement induced by a strong earthquake, which is a key index for the safety evaluation of high core rock-fill dams (H > 200 m). It should be noted that as all four cases have a similar distribution of vertical settlement, only the results from cases A and D are illustrated in Fig. 11. The maximum deformation generally occurs at the crest of the dam. As can be seen from Table 3 and Fig. 11, the traditional approach as employed in case D, which utilizes the average normalized nonlinear soil property curves in equivalent linear analyses, may produce conservative evaluations.

Liquefaction

Liquefaction is another important and interesting design index in geotechnical engineering that has a close relationship with the characteristic of dynamic properties. As far as the high core rock-fill dams (>200 m) are concerned, the liquefaction of the upstream filtering layer is very important for the safety of the entire dam. Seed and Martin (1966) pointed out that if the equivalent cyclic stress ($0.65\tau_{max}$) exceeded the liquefaction resistance, liquefaction at this location would occur. Application of this criterion requires careful attention to the dynamic response results and definition of liquefaction resistance. In this paper, the liquefaction resistance of liquefiable materials is determined according to effective mean stress and equivalent number of cycles (N_{eq} = 30). The degree of the liquefaction of filtering layer for four cases is depicted by the maps in Fig. 12.

Fig. 10. Comparisons of absolute acceleration for the four cases A, B, C, and D referred to in Table 1: (*a*) in the upstream to downstream direction, (*b*) in the vertical direction, (*c*) in the axial direction.

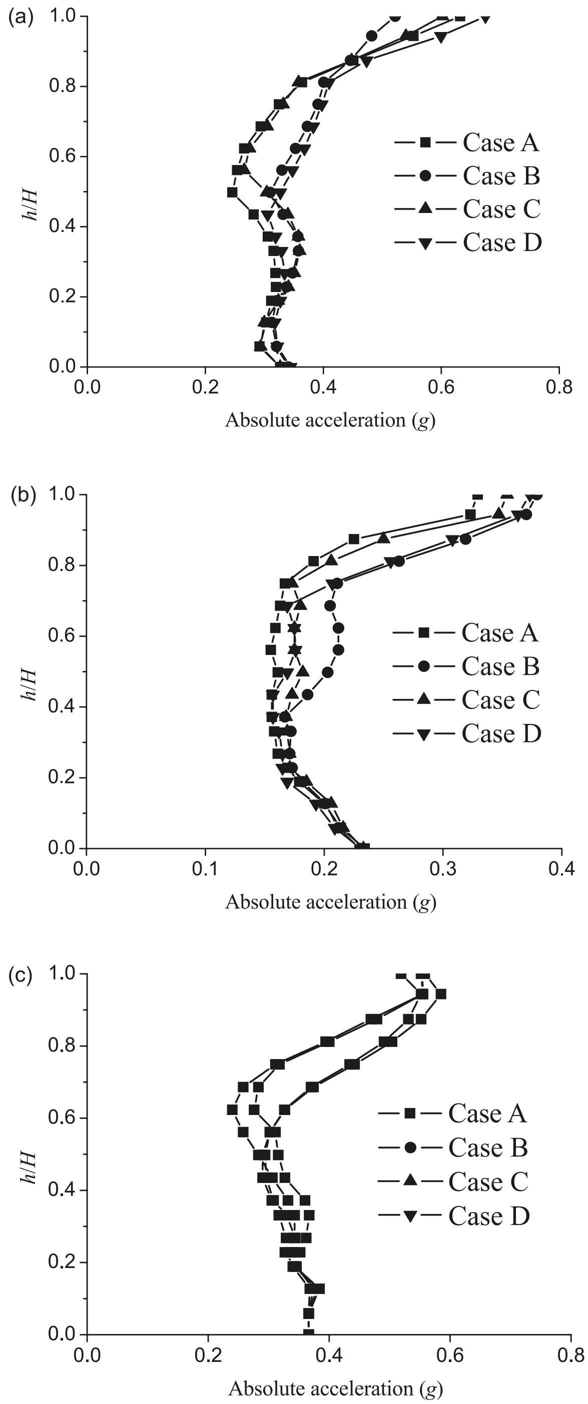
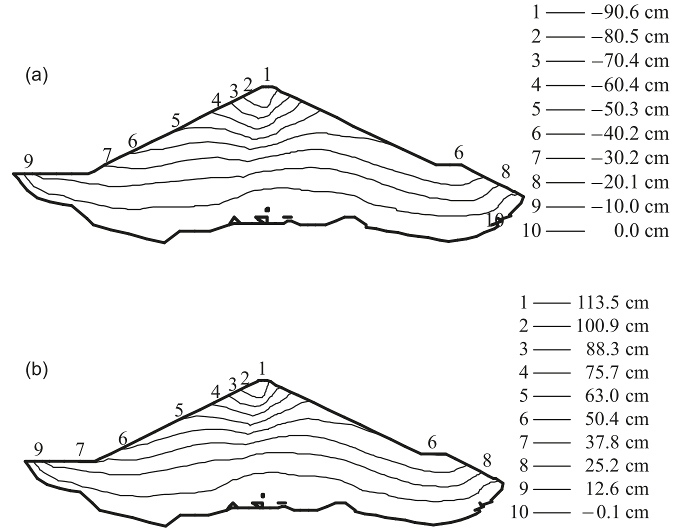


Fig. 11. Distributions of vertical settlement of maximum cross section (*a*) case A, (*b*) case D.



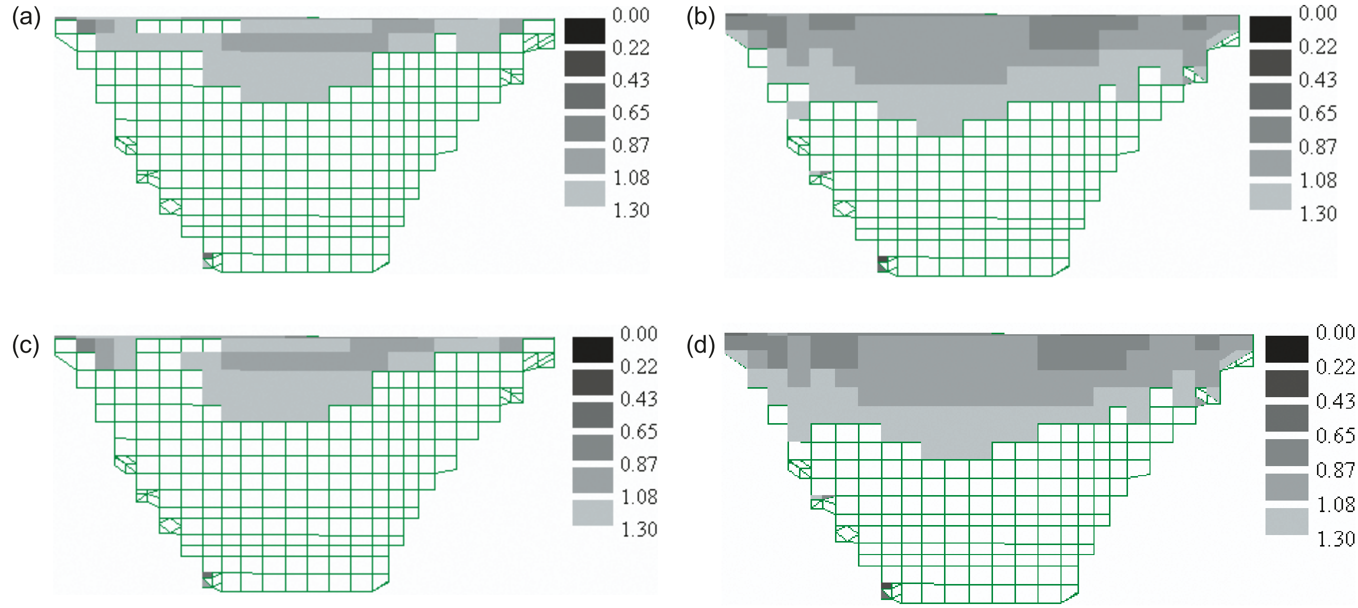
As shown in Fig. 12, the dependency of dynamic property curves on effective confining pressure has a significant influence on the distribution of liquefiable elements in the filtering layer. Furthermore, it can be easily observed that the liquefaction of the filtering layer is more sensitive to the dynamic modulus than to the damping ratio from Figs. 12*a* and 12*c*. Considering that dynamic properties could dramatically alter the liquefaction level, a suitable and refined approach for determination of nonlinear properties in seismic response analyses becomes very important.

Conclusion

Based on the thorough processing of the laboratory data under a wide range of confining pressures ranging from 300 to 2500 kPa for the four filling materials (core, filter, shell, and transitional materials), it is concluded that the effective confining pressure is the main factor controlling the shape of the dynamic property curves G/G_{max} versus γ and ξ versus γ. Also, the average normalized nonlinear property curves could not get a good fit to the data under a wide range of confining pressures as the shearing strain reaches moderate amplitude. However, due to the limitations in the equipment and techniques, some of these problems have been underestimated in the past. In traditional equivalent linear analyses, the average normalized dynamic parameter curves are incorporated into each linear iterative process to obtain the dynamic properties for each soil element corresponding to its current shearing strain. This method is obviously inadequate for seismic analyses of high rock-fill dams (>200 m), which have a broader range of confining pressures.

The traditional approach usually assigns a family of average dynamic property curves for each type of soil material during linear iterative processes to approximate nonlinear behavior. Conversely, the modified approach assigns a set of unique and proper nonlinear soil property curves to each soil element by linearly interpolating or extrapolating according to its current mean effective stress based on the laboratory data under a wide range of confining pressures. The effect of effective confining pressure on dynamic soil properties (presented in terms of normalized shear modulus and material damping curves) has been further quantified based on the data obtained in the laboratory in this refined approach. It is shown that the traditional method used in case D, which utilizes the average nonlinear soil property curves, yields a conservative safety evaluation for high rock-fill

Fig. 12. Sketch of liquefaction in filtering layer in upstream face for four cases referred to in Table 1: (*a*) case A, (*b*) case B, (*c*) case C, (*d*) case D.



dams (>200 m) in terms of absolute acceleration, permanent deformation, and liquefaction.

Acknowledgments

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List of symbols

G secant dynamic shear modulus
G_{max} maximum shear modulus
H maximum height of dam
h distance from crest of dam
K modulus coefficient
N_{eq} equivalent number of cycles
n modulus index
Pa atmospheric pressure
W_D dissipated energy per unit volume in one hysteretic loop
W_S energy stored in an elastic material with the same G as the visco-elastic material
γ shearing strain
ξ damping ratio
σ'_0 confining pressure
τ_{max} equivalent cyclic stress